Higher Dimensional Analogues of Donaldson-Witten Theory.

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Abstract

We present a Donaldson-Witten type field theory in eight dimensions on manifolds with $Spin(7)$ holonomy. We prove that the stress tensor is BRST exact for metric variations preserving the holonomy and we give the invariants for this class of variations. In six and seven dimensions we propose similar theories on Calabi-Yau threefolds and manifolds of G_2 holonomy respectively. We point out that these theories arise by considering supersymmetric Yang-Mills theory defined on such manifolds. The theories are invariant under metric variations preserving the holonomy structure without the need for twisting. This statement is a higher dimensional analogue of the fact that Donaldson-Witten field theory on hyper-Kähler 4-manifolds is topological without twisting. Higher dimensional analogues of Floer cohomology are briefly outlined. All of these theories arise naturally within the context of string theory.

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1 Introduction.

Instantons in four dimensions have proven to be remarkably important objects for our understanding of physics and of mathematics. This fact was reflected in [1], where it was shown how quantum field theory could be used to construct the Donaldson invariants for 4-manifolds. As is well known, instantons play a key role in this story.

In [2] instanton equations in higher dimensional flat space were written down. These were subsequently discussed in [3] where we showed that these equations are very naturally associated with the list of holonomy groups of Ricci flat manifolds (in dimension eight or less).

It is natural to ask whether or not some analogue of the topological quantum field theory of [1] exists on these higher dimensional manifolds. That is the purpose of this paper. In fact the theories we discuss here are an extension of the Donaldson-Witten theory [1] to higher dimensions. This is due to the two basic facts that (i) the correlation functions are invariant under metric deformations which preserve the holonomy structure of the manifold and (ii) the higher dimensional instantons are minima of the Yang-Mills action and can therefore be used to evaluate the observables of the theory.

In the next section we will discuss the instanton equations themselves, and review the relation with certain Ricci flat manifolds. We also introduce some notation which will prove useful later in the paper. In section 3 we introduce the action and show that the observables of the theory are invariant under metric deformations which preserve the holonomy. In section 4 we describe the variational calculus (in the case of the 8-dimensional theory) which is required to verify the formulae of section 3. This calculation also proves useful from another point of view, since we are able to present a very simple (local) discussion of some theorems and results given in [4, 5, 6]. In section 5 we give explicit expressions for the BRST invariant observables from which correlation functions can be constructed. In section 6 we outline the higher dimensional version of Floer theory which, as in [1], arises very naturally in the Hamiltonian formulation.

In our concluding section we point out that for the manifolds we have been discussing these field theories are *not* twisted versions of super-Yang-Mills theory. Rather these theories are simply super-Yang-Mills theories formulated on manifolds with reduced holonomy groups. The theories we discuss here are invariant under a certain class of metric variations, however

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they arise without the need for twisting. This statement is in fact a generalisation of the statement that the four-dimensional theory of [1] formulated on a hyper-Kähler 4-manifold is topological *without* the need for twisting [7].

2 Instanton Equations in $D > 4$.

In this section we give a brief review of the instanton equations in $D > 4$. This will set the notation and conventions for the following sections.

Higher dimensional instanton equations were first written down in [2], for $4 \leq D \leq 8$. As in four dimensions, these equations are first order self-duality equations for gauge fields. The general form of the equations is:

$$\lambda F_{\mu\nu} = \frac{1}{2}\phi_{\mu\nu\rho\sigma}F^{\rho\sigma} \tag{1}$$

This set of equations will be the focus of this paper. In (1), F is the field strength for the gauge field A, and the indices run from 1 to D. In [2] these equations were considered in flat space. However, it is natural to consider these equations on curved manifolds [3]. We assume that the manifold on which the gauge field propagates is a D-dimensional Riemannian manifold. This means that its holonomy is contained within $SO(D)$. The totally anti-symmetric tensor ϕ is a singlet of the holonomy group $H \subset SO(D)$. This can only occur if H is a proper subgroup of $SO(D)$ and not $SO(D)$ itself. Thus, requiring that the instanton equations are non-trivial automatically implies a reduction of the Lorentz group of the theory from $SO(D)$ to H.

It is a problem in group theory to calculate when the instanton equations are non-trivial. This was done in [2]. It was further pointed out in [3] that if F is the Riemannian curvature 2-form for the D-manifold, then the solutions to (1) are precisely the Ricci flat manifolds whose holonomy is given by Berger's classification [8]. In fact for $4 < D \leq 8$, all Ricci flat manifolds with holonomy a proper subgroup of $SO(D)$ admit a covariantly constant 4-form, and ϕ is this 4-form. Thus the instanton equations have the possibility of being non-trivial on any $4 < D \leq 8$ dimensional Ricci flat manifold, with $\phi_{\mu\nu\rho\sigma}$ being given by the components of the corresponding holonomy singlet 4-form. We will mainly be interested in the cases when $D = 8, 7, 6$ and H is $Spin(7), G_2, SU(3)$ respectively.

For $D = 8$ and $Spin(7)$ holonomy, ϕ is a Hodge self-dual 4-form (for a given choice of orientation for the 8-manifold M_8). The instanton equations

are then non-trivial when $\lambda = -1$ and 3. The components of the tensor ϕ are closely related to the structure constants of the octonions, and for a given choice of octonionic structure constants, there exists one choice for ϕ (for precise details on how this works see [9]). We will choose ϕ as in [4] and its components in an orthonormal frame are given by:

$$[1256] = [1278] = [3456] = [3478] = [1357] = [2468] = [1234] = [5678] = 1$$
$$[1368] = [2457] = [1458] = [1467] = [2358] = [2367] = -1 \qquad (2)$$

where $[ijkl]$ means ϕ_{ijkl} and all other components are zero. Since this form is $Spin(7)$ invariant it induces a metric on the 8-manifold M_8. Moreover, if the form is covariantly constant then the holonomy group of the associated metric is $Spin(7)$ [10]. In fact this is so if and only if ϕ is closed [11]. Following [4], we will refer to 4-forms admitting some isomorphism with ϕ above as *admissable $Spin(7)$* structures.

Group theoretically, the instanton equations tell us that F transforms under a certain representation of the holonomy group. Precisely which representation is determined by λ. For the case of $H = Spin(7)$ holonomy, $\lambda = -1$ puts F in a **21** of H, and $\lambda = 3$ gives F as a **7** of H. This is due to the splitting of the adjoint of $SO(8)$ under $Spin(7)$:

$$\mathbf{28} \rightarrow \mathbf{21} + \mathbf{7} \qquad (3)$$

This splitting means that $\Lambda^2(TM)$ decomposes into two orthogonal subspaces of 2-forms. These subspaces consist of 2-forms with 7 and 21 independent components respectively. This means that we can introduce projection operators which project onto the **7** and **21** dimensional pieces of any 2-form. These are given by

$$P_7 = \frac{1}{4}(1 + \frac{1}{2}\phi), \qquad P_{21} = \frac{3}{4}(1 - \frac{1}{6}\phi). \qquad (4)$$

We are using a shorthand matrix notation where, for example, the first expression above represents

$$P_{7\,\mu\nu}{}^{\alpha\beta} = \frac{1}{4}\left(\delta^\alpha_{[\mu}\delta^\beta_{\nu]} + \frac{1}{2}\phi_{\mu\nu}{}^{\alpha\beta}\right). \qquad (5)$$

These projectors satisfy the relations

$$(P_7)^2 = P_7, \qquad (P_{21})^2 = P_{21}$$
$$P_7 P_{21} = P_{21} P_7 = 0, \qquad P_7 + P_{21} = 1. \qquad (6)$$

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Note that the form ϕ satisfies

$$\phi^2 = 4\phi + 12. \tag{7}$$

2.1   D= 7,6

By setting to zero those components of F and ϕ which contain (say) an 8 index, one gets a set of instanton equations in $D = 7$. This corresponds to the reduction of $Spin(7)$ to G_2. Of course, the resulting ϕ is a G_2 singlet. This means we are now considering the equations on a 7-manifold of G_2 holonomy. The instanton equations (1) now correspond to the restriction of F to **14** and **7** dimensional representations of G_2 according to the splitting of the **21** of $SO(7)$ into a **14** and **7**. In this case, ϕ satisfies

$$\phi^2 = 8 + 2\phi. \tag{8}$$

This gives the values of λ in (1) as -1 and 2, corresponding to F being in the **14** and **7** respectively. The normalised projection operators in this case are:

$$P_7 = \frac{1}{3}\left(1 + \frac{1}{2}\phi\right), \tag{9}$$

and

$$P_{14} = \frac{2}{3}\left(1 - \frac{1}{4}\phi\right). \tag{10}$$

By setting to zero one further set of components of F and ϕ, which contain (say) a 7-index, one gets a set of instanton equations in $D = 6$. This corresponds to the reduction of the holonomy group from G_2 to $SU(3)$. In this case, ϕ is proportional to $k \wedge k$, where k is the Kahler form of the Calabi-Yau threefold.

3   The Field Theories

The purpose of this paper is to describe how the moduli space of solutions to (1) can be used to extract quantities (observables) on manifolds with holonomy H (as above) which are invariant under metric deformations which preserve the holonomy. These will follow from a simple generalisation of the field theory in [1], which provided a physical formulation of Donaldson theory.

We will construct theories associated with the moduli space of solutions to (1) using the paradigm of fields, symmetries and equations [12, 13]. This construction will turn out to have a similar structure as that of [1].

As explained in [1] and reviewed in more detail in [13], in topological field theories with gauge symmetries one introduces an anticommuting 1-form which is the fermionic partner of the gauge field. Under the BRST symmetry the gauge field transforms into this 1-form. One further introduces a scalar field which is invariant under the BRST symmetry Q. This scalar, being BRST invariant can then be used to construct topological observables. All fields transform in the adjoint of the gauge group. With this "multiplet", if the fermionic 1-form transforms under Q into a gauge transformation generated by a parameter which is the scalar field, then $Q^2 = 0$, up to gauge transformations. Since $Q^2 = 0$, one can study Q-cohomology. The Q cohomology classes are the observables of the theory. Essentially, this multiplet is associated with the symmetries of the theory.

In addition to the above multiplet one also introduces fields which encode the moduli problem one is interested in. In the case we are interested in, this includes a field which transforms under Q into the instanton equation itself. For example, in $D = 8, 7$, the instanton equation asserts that $P_7 F = 0$ with the appropriate P_7 defined above. We thus include a fermionic 2-form which transforms into $P_7 F$. Finally, it is also useful to introduce two more scalars, with opposite statistics. An action with the above structure in four dimensions was given in [1], and it turns out that a suitable choice of Lagrangian in higher dimensions is provided by a similar Lagrangian to [1], but now considered as a theory defined on a D-dimensional manifold. Of course, in order to define the theory on such a manifold one requires the existence of a 4-form which is a singlet of the holonomy group H. This group must be a proper subgroup of $SO(D)$. In eight and lower dimensions the maximal proper subgroups are $Spin(7)$, G_2 and $SU(3)$ for $D = 8, 7$ and 6.

We will give the Lagrangians for the 8 and 7 dimensional cases explicitly. We expect the 6 dimensional model to take a similar form, with the complex structure playing an important role. In $D = 8, 7$ dimensions we will consider a theory defined on M_D (where M_D has holonomy $Spin(7)$ or G_2 respectively). The field content in these theories will be the same as that in the four dimensional theory in [1], except that the duality conditions on the 2-form will be dimension-dependent relations.

Explicitly, the action for the 8 dimensional theory is given by

$$S = S_1 + S_2, \qquad (11)$$

where

$$
\begin{aligned}
S_1 &= \mathrm{Tr} \int_{M_8} d^8 x \sqrt{g} \{ \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2} \varphi D^\alpha D_\alpha \lambda - i\eta D^\alpha \psi_\alpha \\
&+ 2i(D_\alpha \psi_\beta).\chi^{\alpha\beta} - \frac{i}{4}\varphi[\chi_{\alpha\beta}, \chi^{\alpha\beta}] - \frac{i}{2}\lambda[\psi_\alpha, \psi^\alpha] \\
&- \frac{i}{2}\varphi[\eta, \eta] - \frac{1}{8}[\varphi, \lambda]^2 \},
\end{aligned}
\qquad (12)
$$

and

$$S_2 = \frac{1}{4} \mathrm{Tr} \int_{M_8} d^8 x \sqrt{g} F_{\alpha\beta} \tilde{F}^{\alpha\beta}, \qquad (13)$$

where

$$\tilde{F}_{\alpha\beta} = \frac{1}{2} \phi_{\alpha\beta\gamma\delta} F^{\gamma\delta}. \qquad (14)$$

The action S_2 is the 8 dimensional instanton action [3][4]. In S_1 the commuting fields are the gauge field A_α for which $F_{\alpha\beta}$ is the curvature, and two scalar fields, φ and λ. The anticommuting fields are a 1-form ψ_α, a self-dual 2-form $\chi_{\alpha\beta}$ (which in the $Spin(7)$ and G_2 holonomy cases is in the **7** of the holonomy group) and a scalar, η. Note that the φ which appears in S_1 is a scalar field and should not be confused with the $Spin(7)$ structure (ϕ) which appears in the instanton equations and in S_2.

The action S is invariant under the BRST transformations $\delta = -i\epsilon\{Q, \ \}$, with anticommuting parameter ϵ,

$$\{Q, A\} = -\psi \qquad (15)$$

$$\{Q, \varphi\} = 0 \qquad (16)$$

$$\{Q, \lambda\} = -2\eta \qquad (17)$$

$$\{Q, \eta\} = \frac{i}{2}[\varphi, \lambda] \qquad (18)$$

$$\{Q, \psi\} = -iD\varphi \qquad (19)$$

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[4] In [3] we stated that S_2 is a topological invariant. However our later formulae show that this is not the case.

$$\{Q, \chi_{\alpha\beta}\} = \frac{1}{2}i(F + \tilde{F})_{\alpha\beta}. \tag{20}$$

In the 7 dimensional case, the action is again $S_1 + S_2$ as above, with the following differences: Firstly, one integrates over the 7 dimensional manifold, and utilises the 4-form ϕ appropriate to the G_2 case; secondly, the coefficients of the $(D_\alpha\psi_\beta).\chi^{\alpha\beta}$ and $\varphi[\chi_{\alpha\beta}, \chi^{\alpha\beta}]$ terms are $\frac{3i}{2}$ and $\frac{-3i}{16}$ respectively; finally, the BRST symmetry is again of the form as in the 8 dimensional case, except that the χ variation is now $\{Q, \chi_{\alpha\beta}\} = \frac{2}{3}i(F + \tilde{F})_{\alpha\beta}$.

Apart from factors these actions are of the same form as that in [1], and similarily have an additive "ghost" number symmetry (U), for which the charge assignments are $(0, 2, -2, -1, 1, -1)$ for the fields $(A, \varphi, \lambda, \eta, \psi, \chi)$, respectively.

The Lagrangian for the theories is BRST exact:

$$L = -i\{Q, V\}, \tag{21}$$

where V is given in the 8 dimensional case by

$$V = \frac{1}{2}\text{Tr}\left(F_{\alpha\beta}\chi^{\alpha\beta} + \psi_\alpha D^\alpha\lambda - \frac{1}{2}\eta[\varphi, \lambda]\right), \tag{22}$$

and in the 7 dimensional case by

$$V = \frac{1}{2}\text{Tr}\left(\frac{3}{4}F_{\alpha\beta}\chi^{\alpha\beta} + \psi_\alpha D^\alpha\lambda - \frac{1}{2}\eta[\varphi, \lambda]\right). \tag{23}$$

4 On the Metric Dependence of the Theories.

In order to prove that the correlation functions of a field theory on a manifold M are independent of the metric tensor on M one shows that the energy momentum tensor is Q-exact. This can be done explicitly for the theories we are discussing here if we restrict our attention to manifolds of reduced holonomy and metric deformations which preserve this. Under a small change $\delta g_{\alpha\beta}$ in the metric g on M_D, the action changes by

$$\delta S = \frac{1}{2}\int_{M_D}\sqrt{g}\delta g^{\alpha\beta}T_{\alpha\beta}, \tag{24}$$

which defines the energy-momentum tensor $T_{\alpha\beta}$.

In the eight-dimensional case, when M_8 has $Spin(7)$ holonomy, the energy-momentum tensor is given by the following expression:

$$
\begin{aligned}
T_{\alpha\beta} &= \operatorname{Tr}\{F^\mu_{\ (\alpha}F_{\beta)\mu} + F^\mu_{\ (\alpha}\tilde{F}_{\beta)\mu} - \frac{1}{4}g_{\alpha\beta}F_{\gamma\delta}(F^{\gamma\delta} + \tilde{F}^{\gamma\delta}) \\
&+ 2iD_{[\alpha}\psi_{\sigma]}\chi_\beta^{\ \sigma} + 2iD_{[\beta}\psi_{\sigma]}\chi_\alpha^{\ \sigma} - ig_{\alpha\beta}D_\tau\psi_\sigma\chi^{\tau\sigma} \\
&- D_{(\alpha}\varphi D_{\beta)}\lambda + \frac{1}{2}g_{\alpha\beta}D_\sigma\varphi D^\sigma\lambda + 2iD_{(\alpha}\eta\,\psi_{\beta)} - ig_{\alpha\beta}D_\sigma\eta\,\psi^\sigma \\
&- 2i\lambda\psi_\alpha\psi_\beta + ig_{\alpha\beta}\lambda\psi_\sigma\psi^\sigma + \frac{i}{2}g_{\alpha\beta}\varphi[\eta,\eta] + \frac{1}{8}g_{\alpha\beta}[\varphi,\lambda]^2\}.
\end{aligned} \tag{25}
$$

To derive this expression for $T_{\alpha\beta}$, one needs to know the variations of the holonomy structure ϕ and the self-dual 2-form χ, as their definitions are metric dependent. We will discuss this in some detail later in this section.

One can check that this energy-momentum tensor is Q-exact:

$$
T_{\alpha\beta} = -i\{Q, \lambda_{\alpha\beta}\}, \tag{26}
$$

where

$$
\lambda_{\alpha\beta} = \operatorname{Tr}\{2F_{(\alpha}^{\ \ \mu}\chi_{\beta)\mu} - \frac{1}{2}g_{\alpha\beta}F_{\gamma\delta}\chi^{\gamma\delta} + \psi_{(\alpha}D_{\beta)}\lambda - \frac{1}{2}g_{\alpha\beta}\psi_\sigma D^\sigma\lambda + \frac{1}{4}g_{\alpha\beta}\eta[\varphi,\lambda]\}. \tag{27}
$$

This of course is the key property of the theory, since it implies that correlation functions of BRST invariant observables are independent of suitable variations of the metric tensor on M_8. There are some key points which we would like to clarify here. The existence of the singlet 4-form induces a metric tensor on M_8 (see [4] and the next subsection). This metric tensor is Ricci flat and the above energy-momentum tensor is that which comes from varying the Ricci flat metric associated with the 4-form ϕ. We will show that the correlation functions of Q-invariant observables are invariant under metric deformations which preserve the holonomy of M_8. This can be done by showing that the energy-momentum tensor is Q-exact for these variations.

We will now give the details of the variational calculations required to derive the above energy-momentum tensor. We must first compute the variation of ϕ under variations of the metric.

4.1 Metric Dependence of the Holonomy Structure.

In verifying the above results, one needs to note that the field $\chi_{\alpha\beta}$ satisfies a self-duality condition involving the $Spin(7)$ structure ϕ and the metric tensor $g_{\alpha\beta}$. The form ϕ itself also has a metric dependence as we will see below.

The 4-index tensor ϕ is a singlet of a subgroup H of $GL(D)$. This implies that the holonomy group of the manifold on which ϕ is defined is at most H. In the cases in which we are interested $H \subset SO(D)$ and thus the manifold M_D is Riemannian. The existence of ϕ therefore induces a metric on M_D. Furthermore, every choice of ϕ generically induces a different metric on M_D. This means that given a choice of ϕ (and therefore a metric g), a change in the metric induces a change in ϕ. In this section we will explicitly calculate what this change is. We will restrict our attention to the case in which $D = 8$ and H is $Spin(7)$.

In our conventions ϕ is Hodge self-dual:

$$\phi = *\phi. \tag{28}$$

With respect to the original metric g, there exists a splitting of the space of 4-forms on M_8 into the (orthogonal) spaces of Hodge self-dual and anti-self dual 4-forms. Varying the metric tensor g in the above equation gives a variation of ϕ. We may therefore introduce projection operators which project any 4-form onto its self or anti self-dual components. The equation that ϕ is self-dual may be written as

$$P_+\phi = \phi \tag{29}$$

Varying this equation one obtains

$$\delta P_+.\phi + P_+.\delta\phi = \delta\phi \tag{30}$$

Thus

$$(1 - P_+)\delta\phi = \delta P_+.\phi \tag{31}$$

However,

$$P_+ + P_- = 1 \tag{32}$$

and thus the variation of the ϕ self-duality condition yields

$$P_-\delta\phi = \delta P_+.\phi \tag{33}$$

In other words, the self-duality condition allows one to determine the component of ϕ which varies into the anti-self-dual chamber. In components one obtains that

$$[(\delta P_+).\phi]_{\alpha\beta\gamma\delta} = -\frac{1}{4}(\delta g^{\sigma\tau} g_{\sigma\tau})\phi_{\alpha\beta\gamma\delta} + 2\delta g_{\lambda[\alpha}\phi^{\lambda}{}_{\beta\gamma\delta]} \tag{34}$$

By the above, this expression is Hodge anti self-dual. However, the first term on the right is self-dual because it is proportional to ϕ. Since we have considered arbitrary variations in the metric tensor, we may certainly consider those which are traceless (which would remove this term from the above expression). However, upon further consideration one sees that if one writes the metric variation of the second term on the right as the sum of a trace and traceless part, then the trace term exactly cancels the first term. Thus, the expression above is Hodge anti self-dual as required with arbitrary metric variations. Thus the anti self-dual variation of ϕ under arbitrary variations of the metric is

$$P_-(\delta\phi)_{\alpha\beta\gamma\delta} = 2\delta\tilde{g}_{\lambda[\alpha}\phi^{\lambda}{}_{\beta\gamma\delta]}. \tag{35}$$

where $\delta\tilde{g}_{\alpha\beta}$ is traceless.

Usually, in variational problems of this type in which one has a decomposition of a vector space into two (or more) orthogonal subspaces, in the variation with respect to the metric of a vector which belongs solely to one of the subspaces, one can only determine the variation of the vector into the orthogonal subspace. The orthogonal component of the variation can then only be determined if the original vector satisfies further identities which involve the metric tensor. This is the case for ϕ.

In flat space, ϕ satisfies identities which are given in [9]. These are easily generalised to curved spaces by replacing the flat space metric by the curved one. As these identities are crucial in proving the relations given in this paper, we reproduce them here. The basic one is

$$\phi_{\mu\alpha\beta\gamma}\phi^{\mu\nu\rho\sigma} = 6\delta^{\nu}_{[\alpha}\delta^{\rho}_{\beta}\delta^{\sigma}_{\gamma]} + 9\phi_{[\alpha\beta}{}^{[\nu\rho}\delta^{\sigma]}_{\gamma]}, \tag{36}$$

from which by contractions one deduces that

$$\phi_{\mu\nu\beta\gamma}\phi^{\mu\nu\rho\sigma} = 12\delta^{\rho}_{[\beta}\delta^{\sigma}_{\gamma]} + 4\phi_{\beta\gamma}{}^{\rho\sigma}, \tag{37}$$

$$\phi_{\alpha\mu\nu\rho}\phi_{\beta}{}^{\mu\nu\rho} = 42g_{\alpha\beta}, \tag{38}$$

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and

$$\phi_{\mu\nu\rho\sigma}\phi^{\mu\nu\rho\sigma} = 42.8 \tag{39}$$

The factors in these expressions are the secret of the consistency of our results. It is important to note also that the variation (35) can be checked to be consistent with these relations.

Having calculated the Hodge anti self-dual variation of ϕ, we still require calculating the self-dual variation. In order to proceed with this, one now needs to compare what we have learned above with what is known about the moduli space of $Spin(7)$ structures on M. The space of all possible choices for ϕ (modulo diffeomorphisms isotopic to the identity) was studied in [4]. In particular, it was shown that this moduli space is $b_4^- + 1$ dimensional, where b_4^- is the number of harmonic Hodge anti self-dual four-forms on M. Further, the moduli space of Ricci flat metrics was studied in [5]. This space is also $b_4^- + 1$ dimensional. One would therefore expect that these two spaces are isomorphic. To show this locally let us first compare what we have learned above to the considerations in [5]. In the above we have shown that the anti-self-dual variation of ϕ with respect to the metric tensor of M is given by:

$$(\delta\phi)_{\alpha\beta\gamma\delta} = 2\delta\tilde{g}_{\lambda[\alpha}\phi^\lambda{}_{\beta\gamma\delta]} \tag{40}$$

where the variation of the metric in the above is restricted to be traceless. In [5] it was shown that given a symmetric, *traceless* 2-index object $h_{\lambda\alpha}$ that the 4-index tensor

$$h_{\lambda[\alpha}\phi^\lambda{}_{\beta\gamma\delta]} \tag{41}$$

is Hodge anti self-dual[5].

But this is precisely the form of our above expression for the anti self-dual variation of the $Spin(7)$ structure ϕ. Furthermore it was shown in [5] that the above mapping between anti self-dual 4-forms and symmetric traceless 2-index tensors is invertible and 1-to-1. Thus the space of 4-forms is isomorphic to the space of symmetric, traceless 2-index tensors. It is also true that if the anti self-dual 4-form above is harmonic, then the traceless and divergenceless $\delta\tilde{g}_{\alpha\beta}$ is a zero-mode of the Lichnerowicz equation [5]. This means that changing the $Spin(7)$ structure on the manifold by adding to it a small harmonic anti self-dual 4-form is *equivalent* to volume preserving Ricci flat metric deformations.

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[5]The orientation conventions of [5] are different from ours.

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The key question now is whether or not $\phi' = \phi + \delta\phi$ is a torsion free $Spin(7)$ structure associated with the new Ricci flat metric $g' = g + \delta g$. One can show that it is in two ways. First, since our first order variations are compatible with the identities satisfied by ϕ, ϕ' also satisfies these identities. But these are precisely the identities satisfied by an arbitrary $Spin(7)$ invariant 4-form. Since the result of [5] implies that when g' is Ricci flat ϕ' is closed, we may conclude by theorem 3 of [11] that ϕ' is a torsion free $Spin(7)$ structure. Thus the infinitesimal volume preserving Ricci flat metric variations preserve the holonomy group.

Another way to arrive at the same result is by using the construction of ϕ by parallel spinors [6]. In the second reference of [6] it is shown that under infinitesimal Ricci flat metric variations, a parallel spinor always exists. This means that a covariantly constant, $Spin(7)$ invariant 4-form can always be constructed in the nearby Ricci flat metric.

Apart from these deformations one may also make a scaling of the metric i.e. change the volume. Since we know that the moduli space of $Spin(7)$ structures (modulo diffeomorphisms) has dimension $b_4^- + 1$ [4] and our first order formula for the anti self-dual variation of ϕ has allowed us to identify b_4^- of these variations, we expect that ϕ must scale if we scale the metric tensor. If this were not so the extra coordinate on the moduli space of $Spin(7)$ structures would be missing and our moduli space would only have dimension b_4^-. In general under a transformation of the metric whose traceless part vanishes ϕ will change as follows:

$$\delta\phi = k(\delta g^{\alpha\beta} g_{\alpha\beta})\phi \qquad (42)$$

In order to determine the constant k, one varies the identities which are satisfied by ϕ. This then fixes $k = -1/4$. Adding this part of the variation of ϕ to its anti self-dual variation given in (35) one finds that under an arbitrary change in the metric, the change in ϕ is given by

$$(\delta\phi)_{\alpha\beta\gamma\delta} = 2\delta g_{[\alpha\lambda}\phi^\lambda{}_{\beta\gamma\delta]} \qquad (43)$$

where now the change in the metric can also include variations with non-zero trace. The traceless variations correspond to the addition to ϕ of an anti self-dual 4-form, and the trace variations correspond to scaling ϕ. Following these results we will now restrict all metric variations to those which preserve the holonomy group.

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4.2 Metric Variation of χ.

We may now use what we have learned above to compute the variation of the self-duality condition obeyed by the field χ, which obeys the metric dependent condition:

$$P_7\chi = \chi \tag{44}$$

Varying this equation with respect to the metric tensor on M we obtain

$$\delta\chi = (\delta P_7)\chi + P_7\delta\chi \tag{45}$$

Thus

$$(P_{21})\delta\chi = (\delta P_7)\chi. \tag{46}$$

The above equation means that in the absence of other relations relating χ to the metric tensor, only the variation of χ into the space of 2-forms in the **21** of $Spin(7)$ is constrained. This means that one can consider arbitrary variations of χ in the **7** of $Spin(7)$. Thus in general, the theory above has a continuous family of energy-momentum tensors. However, we find that there is only one choice of this variation for which the energy-momentum tensor is BRST exact, which is

$$(P_7)\delta\chi = \frac{1}{8}(\delta g^{\alpha\beta}g_{\alpha\beta})\chi. \tag{47}$$

The above formula reflects a scaling behaviour of χ under variations of the metric which have non-zero trace.

With this choice, the variation of χ under metric variations is:

$$\delta\chi_{\alpha\beta} = \frac{3}{4}\delta g^{\rho\sigma}g_{\rho[\alpha}\chi_{\beta]\sigma} + \frac{1}{8}\delta g^{\lambda\rho}\phi_\lambda{}^\sigma{}_{\alpha\beta}\chi_{\rho\sigma} + \frac{1}{8}(\delta g^{\sigma\tau}g_{\sigma\tau})\chi_{\alpha\beta}. \tag{48}$$

Using this last formula it is fairly straightforward to compute the energy-momentum tensor, giving equation (25).

The corresponding formulæ to those above for the 7 dimensional case are as follows: The stress tensor is

$$
\begin{aligned}
T_{\alpha\beta} &= \mathrm{Tr}\{F^\mu{}_{(\alpha}F_{\beta)\mu} + F^\mu{}_{(\alpha}\tilde{F}_{\beta)\mu} - \frac{1}{4}g_{\alpha\beta}F_{\gamma\delta}(F^{\gamma\delta} + \tilde{F}^{\gamma\delta}) \\
&+ \frac{3i}{2}D_{[\alpha}\psi_{\sigma]}\chi_\beta{}^\sigma + \frac{3i}{2}D_{[\beta}\psi_{\sigma]}\chi_\alpha{}^\sigma - \frac{3i}{4}g_{\alpha\beta}D_\tau\psi_\sigma\chi^{\tau\sigma} \\
&- D_{(\alpha}\varphi D_{\beta)}\lambda + \frac{1}{2}g_{\alpha\beta}D_\sigma\varphi D^\sigma\lambda + 2iD_{(\alpha}\eta\,\psi_{\beta)} - ig_{\alpha\beta}D_\sigma\eta\,\psi^\sigma \\
&- 2i\lambda\psi_\alpha\psi_\beta + ig_{\alpha\beta}\lambda\psi_\sigma\psi^\sigma + \frac{i}{2}g_{\alpha\beta}\varphi[\eta,\eta] + \frac{1}{8}g_{\alpha\beta}[\varphi,\lambda]^2\}.
\end{aligned}
\tag{49}
$$

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This tensor is Q-exact:

$$T_{\alpha\beta} = -i\{Q, \lambda_{\alpha\beta}\}, \tag{50}$$

where

$$\lambda_{\alpha\beta} = \text{Tr}\{\frac{3}{2}F_{(\alpha}{}^{\mu}\chi_{\beta)\mu} - \frac{3}{8}g_{\alpha\beta}F_{\gamma\delta}\chi^{\gamma\delta} + \psi_{(\alpha}D_{\beta)}\lambda - \frac{1}{2}g_{\alpha\beta}\psi_{\sigma}D^{\sigma}\lambda + \frac{1}{4}g_{\alpha\beta}\eta[\varphi, \lambda]\}. \tag{51}$$

The four form has the same variation as in the 8 dimensional case -

$$(\delta\phi)_{\alpha\beta\gamma\delta} = 2\delta g_{[\alpha\lambda}\phi^{\lambda}{}_{\beta\gamma\delta]}, \tag{52}$$

and the metric variation of the field $\chi_{\alpha\beta}$ is given by

$$\delta\chi_{\alpha\beta} = \frac{2}{3}\delta g^{\rho\sigma}g_{\rho[\alpha}\chi_{\beta]\sigma} + \frac{1}{6}\delta g^{\lambda\rho}\phi_{\lambda}{}^{\sigma}{}_{\alpha\beta}\chi_{\rho\sigma} + \frac{3}{28}(\delta g^{\sigma\tau}g_{\sigma\tau})\chi_{\alpha\beta}. \tag{53}$$

5 Invariants

We may now construct invariants for these theories, following [1]. By these arguments, we deduce that any correlation function of BRST exact objects is zero, and any correlation function of BRST invariant objects is invariant under metric deformations which preserve the holonomy structure of the manifold M. Defining (these functions are in fact independent of the points at which the fields are taken, as in [1])

$$W_0 = \frac{1}{2}\text{Tr}\varphi^2, \tag{54}$$

one finds that

$$
\begin{aligned}
0 &= i\{Q, W_0\} \\
dW_0 &= i\{Q, W_1\} \\
dW_1 &= i\{Q, W_2\} \\
dW_2 &= i\{Q, W_3\} \\
dW_3 &= i\{Q, W_4\} \\
dW_4 &= 0,
\end{aligned}
\tag{55}
$$

with

$$\begin{aligned}
W_1 &= \mathrm{Tr}(\varphi\psi) \\
W_2 &= \mathrm{Tr}(\frac{1}{2}\psi\psi + i\varphi F) \\
W_3 &= \mathrm{Tr}(i\psi F) \\
W_4 &= -\frac{1}{2}\mathrm{Tr}(FF),
\end{aligned} \tag{56}$$

where we understand φ, ψ and F to be zero, one and two forms on the manifold M and there is an implicit understanding of a wedge product in all the above formulae. In addition to the above quantities one can consider a further four quantities which can then be used to define correlation functions. These are the following:

$$W_{i+4} = W_i \wedge \phi \tag{57}$$

for $i = 1, 2, 3, 4$. These are obtainable from equations similar to those above by replacing W_0 with $W_0 \wedge \phi$, since the $Spin(7)$ structure is BRST invariant.

Now, if γ is a k dimensional homology cycle on M then

$$I(\gamma) = \int_\gamma W_k, \tag{58}$$

is BRST invariant by the above relations, and depends only upon the homology class of γ up to BRST exact pieces. Correlation functions of the $I(\gamma)$ are then invariant under metric deformations which preserve the holonomy structure.

6 Floer Formulation

In [1] the relationship between the four-dimensional topological field theory and the Floer cohomology groups of three-manifolds was described. One may ask if there is a relationship between our D-dimensional theories and some cohomological theory in $D-1$ dimensions. In this section we outline such a construction for our eight-dimensional theory. The key to the relationship between Donaldson theory and Floer theory is the existence of a Hamiltonian formulation of the four-dimensional topological field theory on $Y_3 \times R^1$, where

R^1 is the time direction. In a similar manner, we propose that the Hamiltonian formulation of our $D = 8$ theory on $Y_7 \times R^1$ leads to a cohomological theory on Y_7, which in this case is a manifold of G_2 holonomy.

Firstly, we will need the identity

$$D^\alpha(\lambda_{\alpha\beta} + U_{\alpha\beta}) = 0, \qquad (59)$$

where λ is given by eqn. (27), and the antisymmetric tensor $U_{\alpha\beta}$ is

$$\begin{aligned} U_{\alpha\beta} &= \frac{1}{2}\text{Tr}\{-(F_{\alpha\beta} - \tilde{F}_{\alpha\beta})\eta + \phi_{\alpha\beta}{}^{\gamma\delta}\psi_\gamma D_\delta\lambda + [\varphi, \lambda]\chi_{\alpha\beta} \\ &\quad + 2(\tilde{F}_{\alpha\gamma}\chi_\beta{}^\gamma - \tilde{F}_{\beta\gamma}\chi_\alpha{}^\gamma)\}. \end{aligned} \qquad (60)$$

The relatively straightforward proof of this identity involves the use of the equations of motion and the identities given earlier. Now, following [1], for manifolds $M = Y_7 \times R^1$, with Y_7 a compact seven-dimensional manifold of G_2 holonomy, define

$$H = \int d^7x\, T_{00}, \qquad \bar{Q} = 2\int d^7x\, \lambda^{00}. \qquad (61)$$

Then

$$\{Q, \bar{Q}\} = 2H \qquad (62)$$

and

$$[H, \bar{Q}] = 0. \qquad (63)$$

The proof of this last relation mirrors that in section 4 of [1], involving the use of the identity given above for the divergence of $\lambda_{\alpha\beta}$.

The supersymmetry current is given by

$$J_\mu = \text{Tr}\{-\frac{i}{2}\varphi[\psi_\mu, \lambda] + i(F_{\mu\nu} + \tilde{F}_{\mu\nu})\psi^\nu - i\eta D_\mu\varphi - 2iD^\nu\varphi\,\chi_{\mu\nu}\}, \qquad (64)$$

and is conserved, $D^\mu J_\mu = 0$, using the equations of motion. Then we have $Q = -i\int_{Y_7} J^0$. Let indices $i, j, \dots$ run from 1 to 7. Define an operation T which maps

$$\begin{aligned} A_i &\to A_i, & A_0 &\to -A_0, \\ \eta &\to -\psi_0, & \psi_0 &\to \eta, \\ \varphi &\leftrightarrow \lambda, & \psi_i &\to 2\chi_{0i}, \\ \chi_{0i} &\to -\tfrac{1}{2}\psi_i, & \chi_{ij} &\to \tfrac{1}{2}\phi_{ijk0}\psi_k, \end{aligned} \qquad (65)$$

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and which maps $t \to -t$. Then T maps

$$Q \to -\bar{Q}, \qquad \bar{Q} \to Q. \tag{66}$$

It is also straightforward to check that the Hamiltonian is invariant under T. Thus, since $Q^2 = 0$, we have also

$$\bar{Q}^2 = 0. \tag{67}$$

The operators Q and $\bar{Q}$ form the basis for the discussion of the Floer invariants. A similar construction should also work in seven-dimensions where the seven-manifold has the structure $Y_6 \times R^1$ and Y_6 is now a Calabi-Yau threefold.

7   Discussion and Conclusion.

Even though the theories we have discussed here are very much based on the theory in [1], there is a marked difference. Whereas the theory discussed in [1] is a topological quantum field theory on an arbitrary 4-manifold, the theories presented here are only "topological" on manifolds with reduced holonomy groups. This is because the 4-form with which one defines the $D = 4$ instanton exists on all 4-manifolds, but in higher dimensions a 4-form which is a singlet of the holonomy group only exists (at least up to $D = 8$) for the manifolds considered here.

The obvious question which springs to mind is - What is special about this particular list of holonomy groups? At least in physics, these holonomy groups are important because manifolds with these holonomy groups give supersymmetric vacua of string theory and M-theory [14, 15, 16]. One can then ask if there exists any connection between these facts and the theories we have been discussing. We will now argue that a link between string theory and these theories does indeed exist.

7.1   Field Theories From Super Yang-Mills Theory

(Some of the results of this section have been independently obtained in [17]. We are grateful to these authors for discussions.) Let us first discuss the theory which has been the main focus of our paper, the $D = 8$

theory on manifolds of $Spin(7)$ holonomy. By construction, the theory is supersymmetric with a scalar supercharge Q. The theory is not locally supersymmetric, so if there is to be some link with some other theories in eight dimensions it would presumably be another globally supersymmetric theory in eight dimensions. There is only one other such theory in flat space: $D = 8$ super-Yang-Mills theory. With Euclidean signature this theory arises as the effective world-volume theory on a Euclidean Dirichlet 7-brane in Type IIB string theory [18]. In the absence of D-branes Type IIB theory is already a supersymmetric theory, so one natural question to ask is what happens when we consider a Euclidean 7-brane "wrapped" around a manifold of $Spin(7)$ holonomy?

Let us compute the field content of this curved 7-brane theory. We can view the curved theory as a "compactification" of the $D = 8$ super-Yang-Mills theory in flat space. The 8-dimensional Euclidean Lorentz group is $SO(8)$. The bosonic field content of the theory consists of a gauge field (in the $\mathbf{8_v}$) and two scalars. The sixteen fermions are in the $\mathbf{8_s}$ and $\mathbf{8_c}$. The sixteen supercharges have the same $SO(8)$ labels as the fermions. Under the reduction of $SO(8)$ to $Spin(7)$ one gets the following decompositions:

$$\mathbf{8_v} \longrightarrow \mathbf{8} \tag{68}$$

$$\mathbf{8_c} \longrightarrow \mathbf{8} \tag{69}$$

$$\mathbf{8_s} \longrightarrow \mathbf{7} + \mathbf{1} \tag{70}$$

and the two scalars remain as bosonic scalars. Thus the bosonic field content of the theory in curved space consists of two scalars and a gauge field. The fermion field content consists of a scalar, a 1-form and a field in the $\mathbf{7}$ of $Spin(7)$. This is precisely the field content of the $D = 8$ theory we have been discussing. Furthermore, from the sixteen supercharges of the flat space theory we get one scalar supercharge in the curved space theory. This again is reflected in our $D = 8$ theory. Finally, note that $D = 8$ super Yang-Mills theory is essentially a unique theory and thus in considering the theory on a curved manifold one should expect a unique theory with the above field content. We therefore claim that the $D = 8$ theory we have discussed in this paper is just super Yang-Mills theory on a manifold with $Spin(7)$ holonomy. Further evidence to support this claim is that under the $SO(2)$ R-symmetry of $D = 8$ super Yang-Mills the fields will have the same ghost

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numbers as in our theory. The key point however is that one does not have to twist the super-Yang-Mills theory to arrive at the "topological" theory. This contrasts with other topological field theories in four dimensions, however see our comments in the final subsection below.

7.2   D = 7, 6

One can then go on to consider whether or not some similar interpretation as that just given exists for the theories we have discussed on manifolds of G_2 and $SU(3)$ holonomy. There is a unique super Yang-Mills theory in $D = 7$ dimensions. Considered as a theory on a manifold of G_2 holonomy one finds precisely the same field content as we have in the theory with the addition of two scalars: one bosonic and the other fermionic. On closer inspection of the super Yang-Mills action one finds that these "unwanted" scalars would lead to terms in the curved space action which will be Q-invariant. There would also be other terms mixing with the other fields. However, one should also note that the supercharges give rise to two scalar supercharges in the curved space theory, whereas the theory we have been discussing has only one. Setting to zero these unwanted scalar fields may give the field theory we have constructed as a reduction of the $N_T = 2$ theory to $N_T = 1$. The 7-dimensional theory could thus be considered as the theory obtained by "wrapping" a Euclidean 6-brane in Type IIA theory around a manifold of G_2 holonomy. One is free to argue similarly for the case of the $D = 6$ theory on Calabi-Yau threefolds. This theory would then be associated with Euclidean Dirichlet 5-branes in Type IIB theory on a Calabi-Yau threefold. Combining these results with those of [19, 20] gives a unified picture in which in all supersymmetric compactifications of string theory, D-branes wrapped around the supersymmetric cycles give "topological" world-volume theories. Of course, the restriction in the theories we have discussed here to metric variations which are Ricci flat is also very natural from the string theory point of view. This is because non-Ricci flat metrics do not provide a solution to the low energy field equations in superstring theories.

In fact in the work of [19, 20], the supersymmetric 4-cycles in 7 or 8 manifolds of exceptional holonomy are shown to be calibrated submanifolds [21] where the calibration is the four form ϕ that we have discussed in this paper. The calibrated submanifolds are unique in each homology class of the manifold and are precisely the four submanifolds for which ϕ is the volume

form. These submanifolds are those about which one can wrap D-branes without breaking all supersymmetry. The theory of calibrated submanifolds therefore involves an interesting interplay between geometry and topology. The theories that we have discussed in this paper reflect this interplay and thus should be of importance also in the mathematical study of calibrated submanifolds.

7.3   Relation to Heterotic and Type I String Theory.

There is a further possible relation to the heterotic and Type I string theories. In these theories the super-Yang-Mills multiplet explicitly appears in the low energy dynamics. Thus when these theories are compactified on manifolds with the holonomy groups we have discussed, a sector of the theory on the internal manifold could plausibly be describable by the theories we have presented. This is perhaps not surprising given the relation to D-branes we have just discussed. The reason for this is that non-perturbative dualities under which certain Type II and M-theory vacua are believed to be equivalent to certain other heterotic/Type I vacua, often map the gauge symmetries associated with D-branes to those in the heterotic and Type I theories. Dualities between heterotic/TypeI/TypeII and M-theory compactifications on manifolds with exceptional holonomy have been discussed in [15, 16]. We believe that the relationship between the field theories discussed here and heterotic and Type I strings deserves much further study [22].

7.4   A Further Relation With Donaldson-Witten Theory.

As we discussed above, there is one contrast between the theories discussed here and that of [1]: it is not possible to formulate these theories on arbitrary D-dimensional manifolds. Remarkably these theories still turn out to be "topological", in the sense discussed above, even though they only exist on manifolds with reduced holonomy groups. One can ask if this property is reflected in Donaldson-Witten theory.

We showed in [3] that the curvature 2-form for manifolds with holonomy $Spin(7)$, G_2, $SU(3)$ and $SU(2)$ satisfies the instanton equations in higher dimensions. The first three groups are the holonomy groups for the manifolds

on which we have defined the theories discussed here. The last group in this list is the holonomy group of hyper-Kähler manifolds in four dimensions. Given our preceding comments one can then ask, what are the properties of the Donaldson-Witten theory on a hyper-Kähler manifold? As is well known, the theory in [1] is a twisted version of $N = 2$ Super-Yang-Mills theory. However if one considers the theory on a manifold with $SU(2)$ holonomy one finds that the theory after twisting is the same as that before twisting [7]! This is due to the fact that the four-dimensional Lorentz group is $SU(2) \times SU(2)$, but when the manifold is hyper-Kähler one of the $SU(2)$ factors acts trivially on all fields. Thus this is another key property of the four-dimensional theory which is reflected in higher dimensions. The fact that the theories discussed above on reduced holonomy manifolds are topological (in the sense discussed here) without twisting stems from the fact these manifolds admit spinors which are singlets of the holonomy group.

Note Added: Whilst we were writing this paper, the work [23] appeared. Amongst other results, this paper discusses the gauge-fixing of the action S_2 in a number of cases, and makes several similar observations to our work. The proof that these theories are invariant under metric deformations preserving the holonomy structure however requires the analysis of the BRST exactness of the stress tensor which we have presented here. In addition related work with a more mathematical focus has been brought to our attention [24].

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